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THE INFLUENCE OF ACTUATION MODELING ON THE ASSESSED JOINT REACTIONS IN BIOMECHANICAL SYSTEMS

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Abstract

Introduction. Human body biomechanical models are actuated either by net torques at the joints or individual muscle forces whose action around the joints results, by principle, in the net torques. In the model-based inverse dynamics simulation of human movements the assessed joint reactions depend substantially on the choice of the actuation model, which is discussed in the paper. ***Material and methods.*** Using the two actuation models, variant biomechanical models of the lower limb, decomposed from the whole human body, were developed. They were then used for the inverse dynamics simulation of a recorded one-leg jump on the force platform to assess time variations of controls (either net torques or muscle forces) and joint reactions. ***Results.*** The assessed joint reactions obtained using the model actuated by net torques are substantially different from those obtained by means of the model actuated by muscle forces. ***Conclusion.*** The joint reactions computed using the model actuated by net torques do not involve contribution of the tensile muscle forces to the internal loads, and they are therefore underestimated. Determination of joint reactions should thus be based on musculoskeletal models actuated by the muscle forces.

Key words: inverse dynamics simulation, actuation models, internal loads

Introduction

Inverse dynamics simulation is the prevailing non-invasive method for assessment of muscular enforcement and joint internal loads during human movements. The analysis is based on human body models and measured kinematic characteristics of the movements. There are two basic models of actuation of the developed musculoskeletal systems [1-3]: the *deterministic model of actuation* which involves net torques that represent the resultant muscle action at the skeletal joints (Fig. 1a), and the *nondeterministic model of actuation* in which the individual muscle forces are directly used (Fig. 1b). The input data for the inverse simulation are video-recorded kinematic characteristics of the movement and, in the case at hand, measured ground reactions. The results include the time variations of actuation, respectively the net torques or muscle forces required for the movement reproduction, and, in addition, the internal joint reactions.

The analyzed movement (Fig. 2) consists of a short flight phase (hurdle from one leg to another), one-leg contact phase with the ground (landing and take-off), and another flight phase (ballistic flight after the take-off). The lower limb that contacts the ground (here the right leg) is isolated from the whole human body and modeled as a planar kinematic structure composed of b=3 rigid segments: thigh, shank and foot, whose motion is assumed to be performed in the sagittal plane. The segments are interconnected in the knee and ankle joints (ideal hinge joints), and the chain is then attached in the hip joint (another hinge joint) to a moving platform (trunk). The motion of the hip joint is assumed to be known from measurements, and as such the lower limb model has k=3 degrees of freedom. The external loads include the gravity forces and, during the contact phase, reactions from the ground (measured from the force platform).

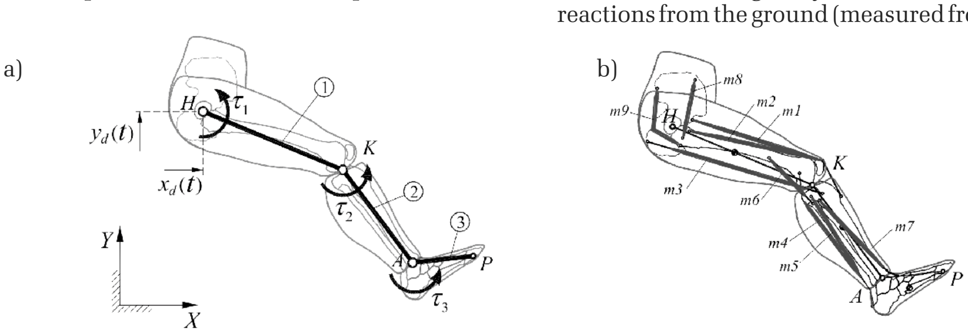

Figure 1. The lower limb models of actuation: a) deterministic, b) nondeterministic

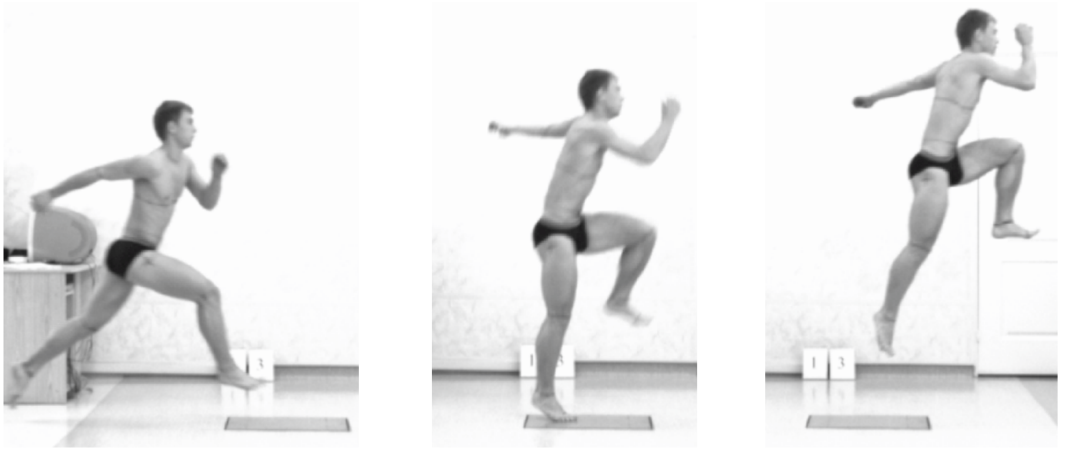

Figure 2. The analyzed movement: one-leg landing and take-off from the force platform

Material and methods

Dynamic equations of motion

The dynamic equations of the lower limb can conveniently be introduced in $n=3b=9$ absolute coordinates $p=[x_{C1}\ y_{C1}\ \theta_1\ x_{C2}\ y_{C2}\ \theta_2\ x_{C3}\ y_{C3}\ \theta_3]^T$ that specify the locations of the mass centers and the angular orientations of the three segments with respect to the inertial frame *XY*. The generic matrix form of the dynamic equations is [4, 5]

$$M\ddot{p}=f_g-C^T(p)\,\lambda+A_r(p)\,\lambda_r+f_a \tag{1}$$

where $M\ddot{p}=f_g$ are the combined dynamic equations of unconstrained segments, with the generalized mass matrix $M=\text{diag}(m_1, m_1, J_{C1}, m_2, m_2, J_{C2}, m_3, m_3, J_{C3})$ and the generalized force vector due to the gravitational forces $f_g=[0 -m_1\,g\ 0\ 0 -m_2\,g\ 0\ 0 -m_3\,g\ 0]^T$, and where m_i and J_{Ci} are the masses and the central moments of inertia of the segments, and g is the gravity acceleration. The generalized force vector $f_g=-C^T\lambda$ results from $l=6$ constraints $z=\Phi(p)=0$ imposed on the segments due to their connections in the joints, and $C=\partial\Phi/\partial p$ is the $l\,x\,n$ (6 x 9) Jacobian matrix. Since all the joints are modeled as revolute ones, l openconstraint coordinates $z=[z_1 \ldots z_6]^T$ exspress the prohibited *X* and *Y* relative translations in the joints, and the corresponding Lagrange multipliers $\lambda=[\lambda_1 \ldots \lambda_6]^T$ are the *X* and *Y* components of the joint reaction forces, both shown in Figure 3. During the contact phase, the external reaction exerted on the foot (measured from the force platform) is reduced to the ankle joint *A*. This reaction has three components $\lambda_r=[R_x\ R_y\ M_A]^T$ irrespective of the contact scenario (Fig. 4), where R_x and R_y are the *X* and *Y* components of the reaction force, and M_A is the reaction force moment with respect to *A*. The ground reaction components are involved in the dynamic equations as $f_r=A_r\lambda_r$, where A_Y is the 9 x 3 matrix of distribution of λ_r in directions of *p*. The explicit formulation of A_r, which has nonzero entries only in the last three rows corresponding to the foot absolute coordinates, is a standard and evident task. Finally, f_a in Eq. (1) is a vector of actuation which is defined in the sequel.

Deterministic and nondeterministic models of actuation

The deterministic model of actuation involves $k=3$ torques $\tau=[\tau_1\ \tau_2\ \tau_3]^T$ that represent the resultant muscle action at joints *H, K* and *A* (Fig. 1a), which explicitly actuate the three degrees of freedom of the system. In the nondeterministic model of actuations, the torques are replaced with the action of $m=9$ forces of muscles (Fig. 1b), and the muscle stresses $\sigma=[\sigma_1 \ldots \sigma_9]^T$ are used as control parameters, $\sigma_i=F_j/A_j$, where F_j is the *j*th muscle force and A_j is physiological cross sectional area, $j=1,\ldots,9$. The generalized force vector f_a in Eq. (1), respectively for the deterministic *(d)* and nondeterministic *(n)* model of actuation, is then modeled as

$$f_a^{\,d}=B_\tau\tau;\ f_a^{\,n}=B_\sigma\sigma \tag{2}$$

where the control distribution matrices B_τ and B_σ are of dimensions $n\,x\,k$ (9 x 3) and $n\,x\,m$ (9 x 9), respectively. Note that the two models of actuation are not equivalent, that is $f_a^{\,d}\neq f_a^{\,n}$ ($B_\tau\tau \neq B_\sigma\,\sigma$). Evidently, the muscle forces in the nondeterministic model must, by principle, result in the same con-

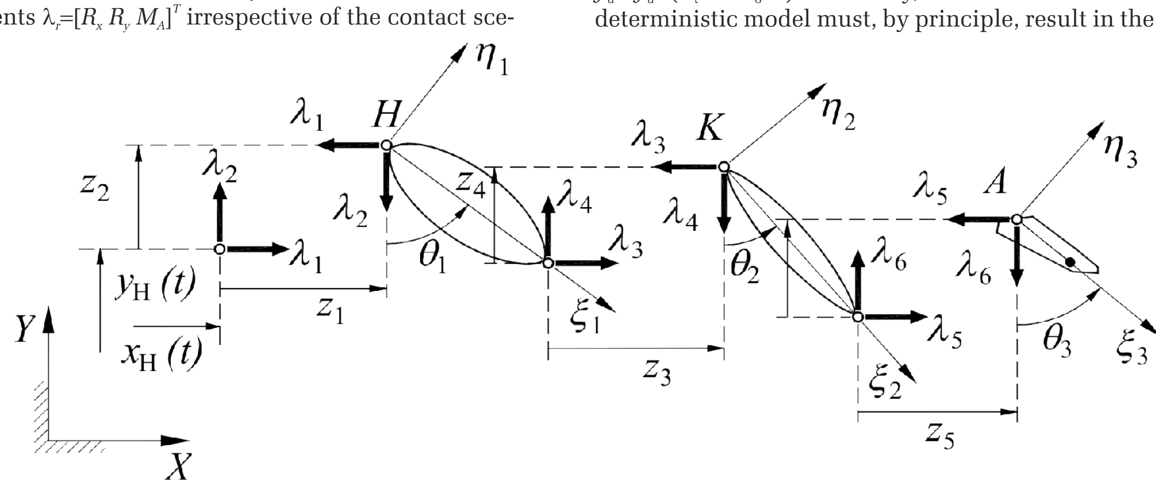

Figure 3. The kinematical chain of the right lower limb, the open-constraint coordinates, and the respective reaction forces

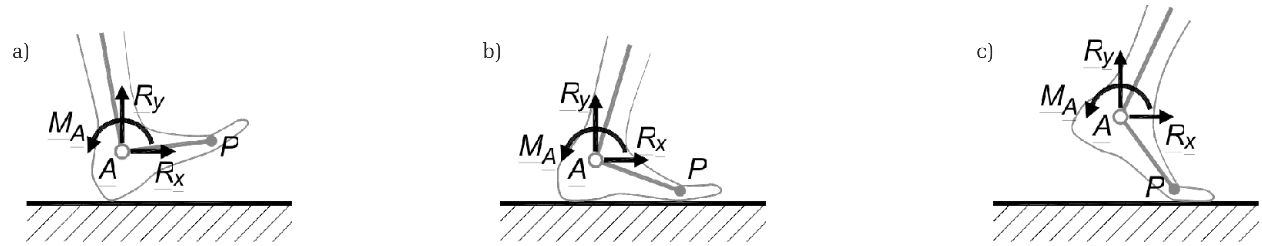

Figure 4. Possible scenarios of the foot contact with the ground, and the components of the external reactions on the foot reduced to the ankle joint *A*

trol torques at the joints as those used in the deterministic model. However, the tensile muscle forces contribute also to the internal joint reactions, which are omitted when the deterministic model of actuation is used. Consequently, in Eq. (1), $f_c^{\,d}=-C^T\lambda^d$ is different from $f_c^{\,n}=-C^T\lambda^n$, and as such $\lambda^d \neq \lambda^n$, where λ^d and λ^n denote the reactions λ when the deterministic and nondeterministic are used, respectively.

The formulation of B_τ is obvious. It is a constant sparse matrix with the entries equal to either 1, –1 or 0, and the nonzero entries are gathered only in the rows related to θ_1, θ_2 and θ_3. The formulation of B_σ is more challenging. Each muscle must be individually modeled concerning its anatomical origin and insertion points. The muscle paths relative to the skeleton must also be identified to determine the actual muscle moment arms about the joints [4, 6, 7]. In the neighborhood of the joints, the muscles or their tendons may wrap on joint capsules, bones and other muscle bellies, and a certain moment arm with respect to the joint is reached irrespective of the relative configuration of the adjacent segments. Consequently, the *effective* origin and insertion points need to be estimated, in which the muscle forces are applied to the segments, and the *effective musculotendon paths* are then defined as lines connecting the effective attachment points. All these issues allow for an explicit formulation of $B_\sigma(p)$ in a way described in [5] (and in [4] for the upper limb), which was followed in the present study.

Anthropometric data

The anthropometric data used in the developed model of a gymnast include: the lengths of the segments, the locations of their mass centers in the local coordinate frames, and then their masses and mass moments of inertia with respect to the mass centers. The lengths of the segments, that is the distances *HK*, *KA* and *AP* between the joints (Fig. 1), were directly measured on the subject, together with the height and weight of the subject. The masses, the mass center locations, and the mass moments of inertia of the segments were then estimated using the regression equations reported in [1, 2, 8], which was concerned with a series of additional measurements of characteristic circumferences and segment lengths of the subject body. The additional data, required for the nondeterministic model of actuation, were the physiological cross sectional areas of the muscles together with all the information required for the modeling of the muscle action – the origin and insertion points of the muscles, and the muscle paths relative the skeleton to determine the actual muscle moment arms about the joints [4, 5, 6, 7]. In this study the data were estimated following the methodology described in [2, 9, 10]. The relevant anthropometric data (for the whole sagittal-plane model of the body), estimated for a 20-year old male subject of mass 76 kg and height 179 cm (seen in Fig. 2), are provided in [11]. The data used in this study were extracted from the reference.

Kinematic data

The analyzed movement (Fig. 2) was video-recorded at 100 frames per second, and the *XY* coordinates of four base points, marked on the subject body at *H, K, A* and *P* (Fig. 1), were digitalized from the photographic images using the direct linear transformation method [12]. With $r_v=[x_v\ y_v]^T$, $v=H, K, A, P$, the position of each of the three segments in the sagittal plane was explicitly defined by two appropriate base points, from which the segment absolute coordinates $p_i=[x_{Ci}\ y_{Ci}\ \theta_i]^T$, $i=1,2,3$, were determined. The raw kinematic data obtained this way, $p(t)$, were finally smoothed to $p_d(t)$ using a second order Butterworth filter [1] with cut-off frequency of 10 Hz. The characteristics $\ddot{p}_d(t)$ were then computed from $p_d(t)$, sampled with the time interval $\Delta t=0.01$ s following the formula

$$\ddot{p}_k=\frac{p_{k+1}-2p_k+p_{k-1}}{\Delta t^2} \qquad (3)$$

Determinate inverse dynamics problem

Applying the deterministic model of actuation in Eq. (1), $f_a^{\,d}=B_\tau\tau$, one can augment the $n \times (l+k)$ (9 x 9) invertible matrix $W(p)=[-C^T(p) \vdots B_\tau]$ so that, using the kinematic characteristics $p_d(t)$ and $\ddot{p}_d(t)$, and the measured ground reactions $\lambda_{rd}(t)$ (during the flying phases $\lambda_{rd}=0$), the determinate inverse dynamics problem can be stated as

$$\begin{bmatrix}\lambda^d\\ \tau\end{bmatrix}=W^{-1}(p_d)[M\ddot{p}_d-f_g-C_r^{\,T}(p_d)\,\lambda_{rd}] \qquad (4)$$

As a solution to Eq. (4), the time variations of joint reactions and muscle torques at the joints during the analyzed movement are explicitly determined, $\lambda^d(t)$ and $\tau(t)$. While the variation $\tau(t)$ obtained this way is suitable in the sense of the modeling assumptions, the variation $\lambda^d(t)$ should be considered defective since the contribution of the tensile muscle forces to the internal loads is omitted in the deterministic formulation. An improved assessment of joint reactions should therefore be based on the nondeterministic model of actuation. Formulation of the indeterminate inverse dynamics problem, in which the time variations of $\sigma(t)$ and (improved) $\lambda^n(t)$ are found, requires some additional manipulations.

Projected dynamic equations

Let us consider the dynamic equations of the lower limb model, introduced in Eq. (1), with the nondeterministic model of actuation, $f_a^{\,n}=B_\sigma\sigma$. While the equations are formulated in $n=9$ absolute coordinates $p=[x_{c1}\ y_{c1}\ \theta_1\ x_{c2}\ y_{c2}\ \theta_2\ x_{c1}\ y_{c1}\ \theta_1]^T$, it is desirable (for the purpose stated below) to project them in the directions of $k=3$ independent coordinates $q=[\theta_1\ \theta_2\ \theta_3]^T$ and the directions of $l=6$ open-constraint coordinates $z=[z_1 \ldots z_6]^T$, both seen in Figure 3. As it was discussed in [13, 14], the relation-

$$p=g(q,z)+\eta(t) \qquad (5)$$

ship expresses the augmented form of joint constraint equations given explicitly, which becomes the standard form $p=g(q,z)+\eta(t)$ after setting $z=0$, and where $\eta(t)$ is consequent to the motion of *H* joint (known from measurements), $x_H(t)$ and $y_H(t)$. For the case at hand the relationship of Eq. (5) is straightforward [4, 5, 11, 13, 14], and is not reported here for brevity. The differentiation of Eq. (5) with respect to time leads then to where the matrices *D* and *E* are of dimensions $n \times k$ (9 x 3) and

$$\dot{p}=\left(\frac{\partial g}{\partial p}\right)\Bigg|_{z=0}\dot{q}+\left(\frac{\partial g}{\partial p}\right)\Bigg|_{z=0}\dot{z}+\dot{\eta}(t)=D(q)\dot{q}+E\dot{z}+\dot{\eta}(t) \qquad (6)$$

$n \times l$ (9 x 6), respectively, and E is a simple constant matrix with the entries equal to either 0 or 1; see [4, 5, 14] for the details.

As shown in [13], the $n \times k$ matrix D is an orthogonal complement matrix to the $l \times n$ constrain matrix C defined in Eq. (1), that is

$$C\,D=0 \;\Leftrightarrow\; D^T C^T=0 \tag{7}$$

The $n \times l$ matrix E has then the features of a pseudoinverse matrix to C, i.e.

$$C\,E=I \;\Leftrightarrow\; E^T C^T=I \tag{8}$$

where I is the $l \times l$ identity matrix.

Projection of the lower limb dynamic equations into the directions of q and z is equivalent, respectively, to premultiplying Eq. (1) with D^T and E^T. Taking into account Eqs. (7) and (8), the projections lead to

$$D^T M\ddot{p} = D^T f_g + D^T A_r(p)\,\lambda_r + D^T b_\sigma \sigma \tag{9}$$

$$E^T M\ddot{p} = E^T f_g - \lambda + E^T A_r(p)\,\lambda_r + E^T b_\sigma \sigma \tag{10}$$

Indeterminate inverse dynamics problem

The dynamic equations projected into q (and τ) directions, given in Eq. (9), provide $k=3$ relationships that describe the contribution of the $m=9$ muscle forces to the $k=3$ net torques at the joints H, K and A. Using $p_d(t)$, $\ddot{p}_d(t)$ and $\lambda_{rd}(t)$, the indeterminate inverse dynamics problem, called muscular load-sharing problem in biomechanics [2, 3], can be stated as the following optimization scheme

$$\begin{cases} \text{minimize} & J(\sigma) \\ \text{subject to} & D^T(p_d)\, B_\sigma(p_d)\sigma = D^T(p_d)\,[M\ddot{p}_d - f_g + A_r(p_d)\lambda_{rd}] \\ \text{and} & \sigma_{\min} \le \sigma \le \sigma_{\max} \end{cases} \tag{11}$$

where J is a chosen cost (objective) function, and $\sigma_{\min}$ and $\sigma_{\max}$ are the physiologically allowable minimal and maximal values of muscle stresses. In this way the individual muscle efforts in the analyzed movement are estimated, here $\sigma_d(t)$. The cost function J used in this study is that proposed by Crowninshield and Brand [15] that minimizes the sum of cubed muscle stresses

$$J(\sigma)=\sum_{j=1}^{m} \sigma_j^3 \tag{12}$$

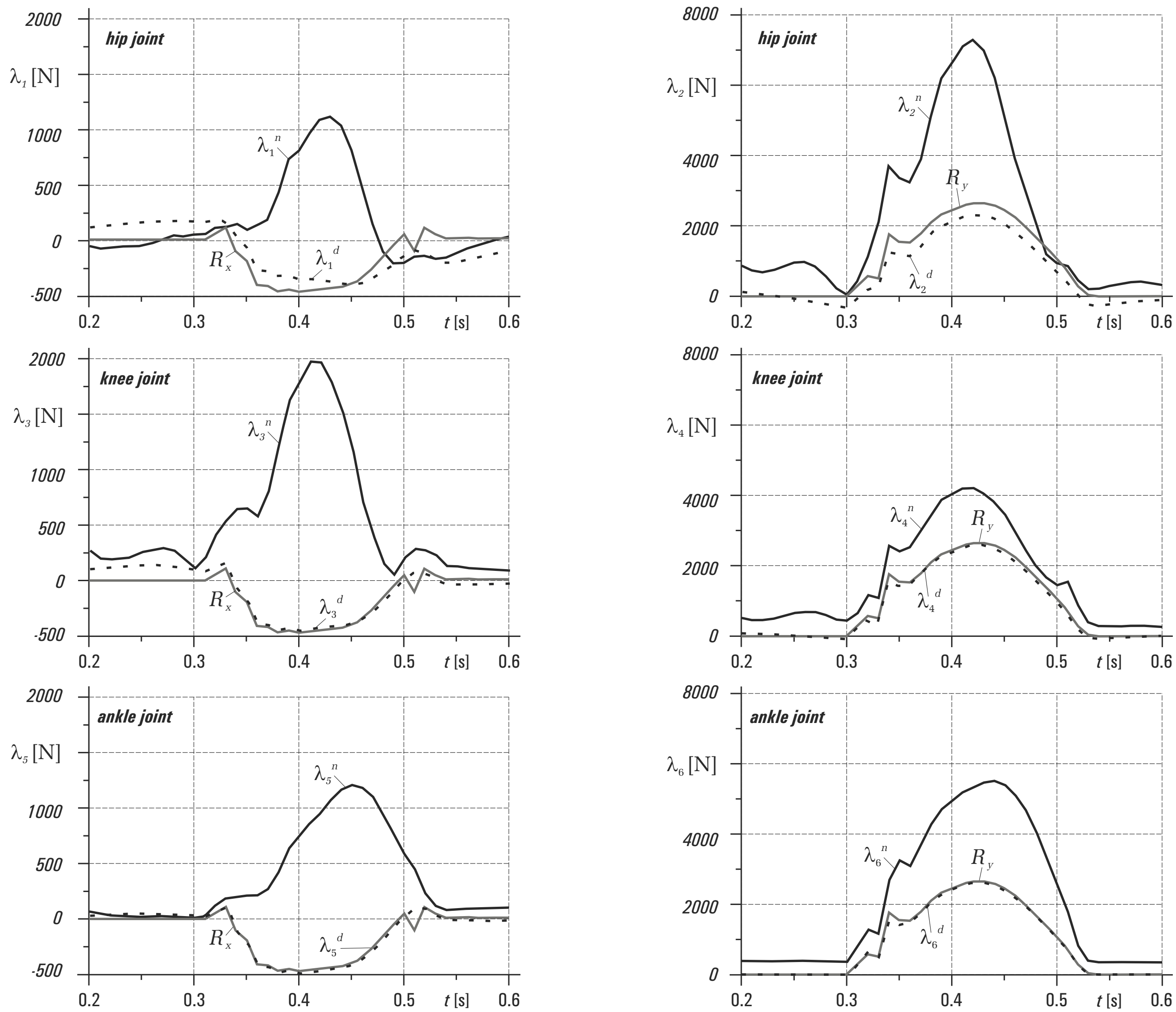

Figure 5. Reaction forces in the model joints obtained using the deterministic and nondeterministic models of actuation, versus the measured ground reactions (in grey): left column – the horizontal reaction components, and right column – the vertical reaction components (please notify the different scales in the graphs for the horizontal and vertical reaction components)

Improved determination of joint reactions

With the application of the nondeterministic model of actuation, using $p_d(t)$, $\ddot{p}_d(t)$ and $\lambda_{rd}(t)$, as before, and then the estimated solution $\sigma_d(t)$ to the above indeterminate inverse dynamics problem, the improved formula for determination of joint reactions results from Eq. (10), which is manipulated to

$$\lambda_d^{\,n}(t) = E^T[f_g + A_r(p_d)\,\lambda_{rd} + B_\sigma(p_d)\sigma_d - M\ddot{p}_d] \qquad (13)$$

By contrast to the internal reactions $\lambda_d^{\,d}(t)$ obtained from Eq. (4), the present solution $\lambda_d^{\,n}(t)$ takes into account the contribution of the tensile muscle forces to the internal loads.

Results

Figure 5 shows the simulation results related to the internal joint reactions , which were estimated using the deterministic and nondeterministic models of actuation, respectively. They are then confronted with the measured ground reaction components R_x and R_y (Fig. 4). During the contact phase, $0.3\ s<t<0.52\ s$, the joint reactions arising from the determinate inverse dynamics problem are almost the same as the applied ground reaction forces, and the small differences are due to the inertial forces of the segments and smoothed kinematic data used in calculations. The inclusion of the contribution of the tensile muscle forces to the internal loads, achieved in the indeterminate inverse dynamics problem, results in substantial enlargement in the reaction forces. As concerns the horizontal components of the reactions, there is also a qualitative difference – the reaction components alter from negative (deterministic model) to positive (nondeterministic model), which is due to the contribution of the tensile muscle forces to the internal loads.

Conclusion

As shown in the paper, an advantage of using the deterministic model of actuation (by means of net torques at the joints) in biomechanical systems is the simplicity in both the mathematical modeling and the inverse dynamics simulation scheme. The vital drawback of the approach is, however, that the internal joint reactions arising from the determinate inverse dynamics problem are underestimated since the contribution of the tensile muscle forces to the internal loads is not involved. Determination of joint reactions should therefore be based on musculoskeletal models actuated by muscle forces, which is related to much more challenging modeling and computational issues.

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